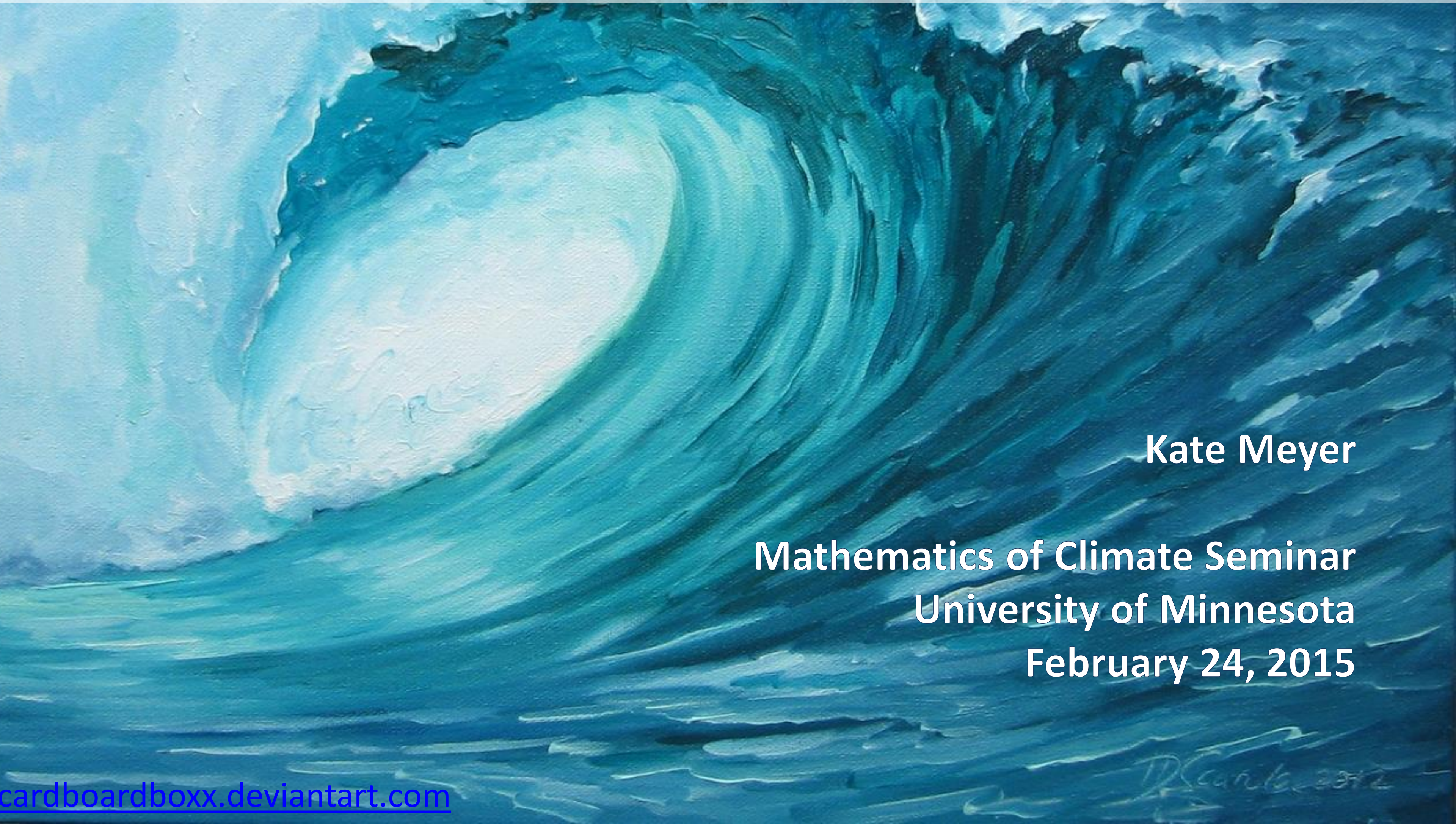
Perspectives on Resilience Using Stommel's Ocean-Box Model



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Mathematics of Climate Seminar
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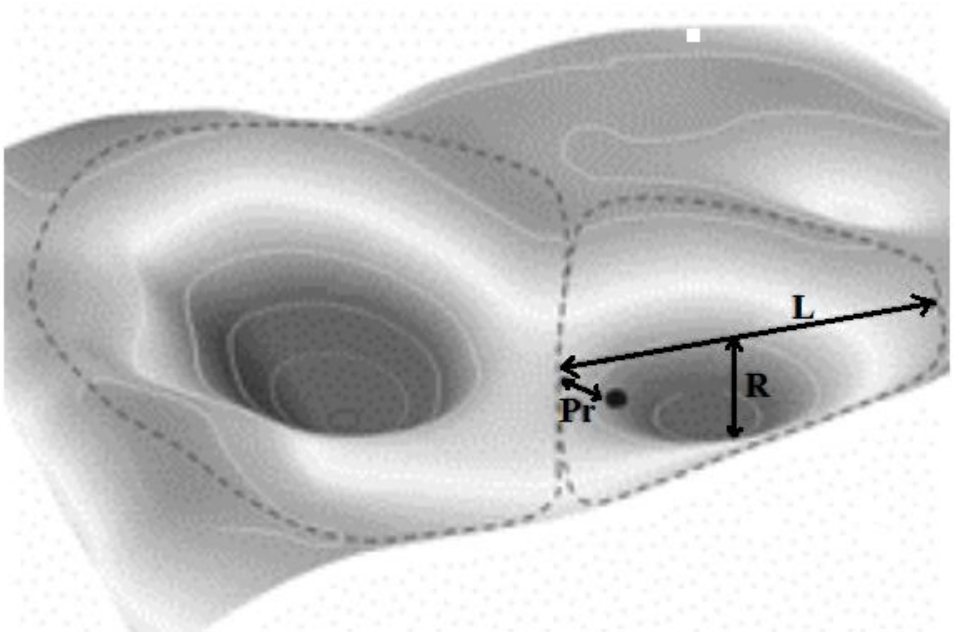
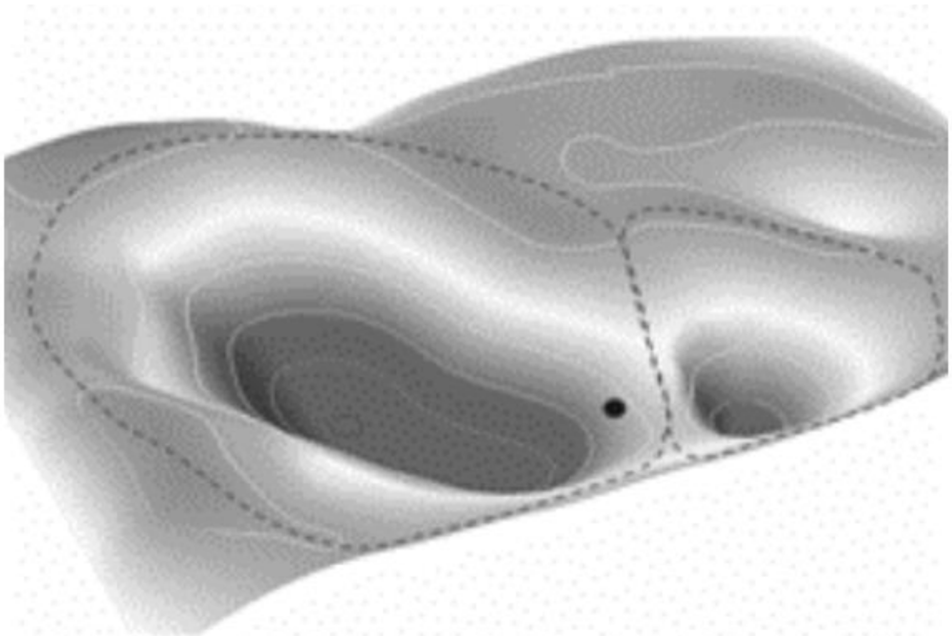
resilience:

the capacity of a system to absorb disturbance and maintain its basic structure and function state variable perturbations

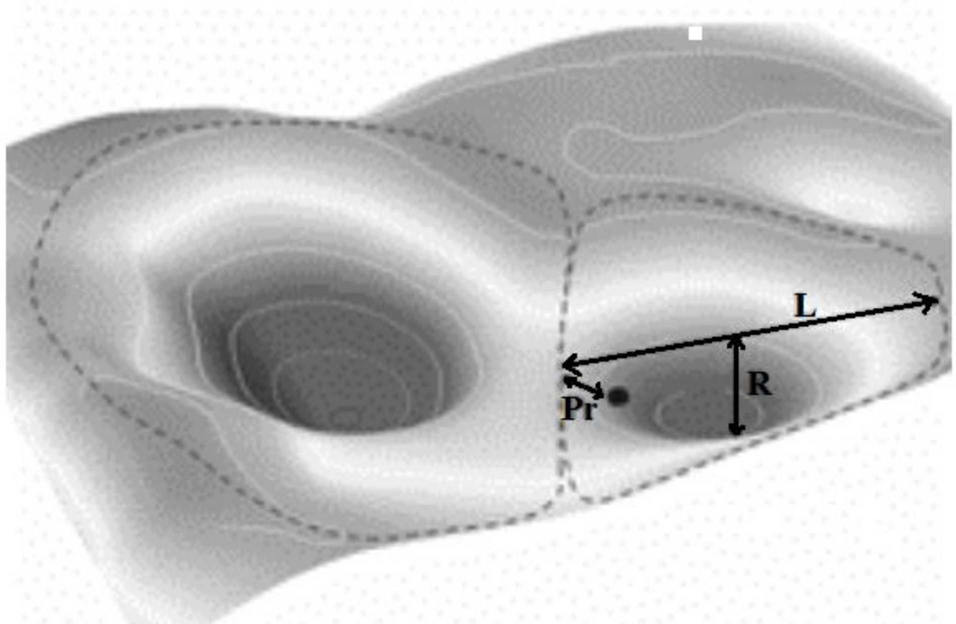


parameter perturbations

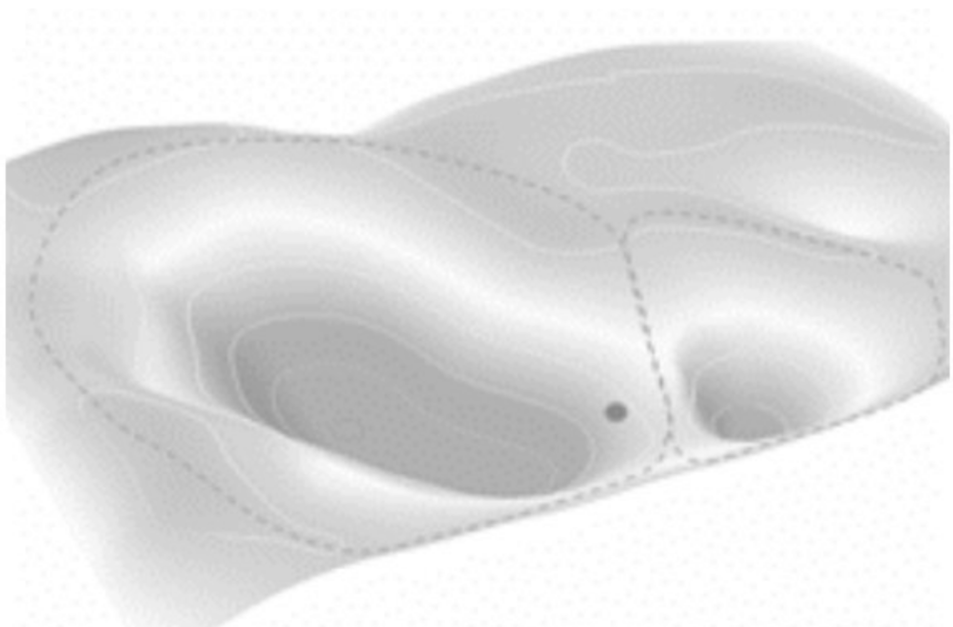


Walker et al. (2004)

state variable perturbations



parameter perturbations



Walker et al. (2004)

$\phi : X \circlearrowleft$

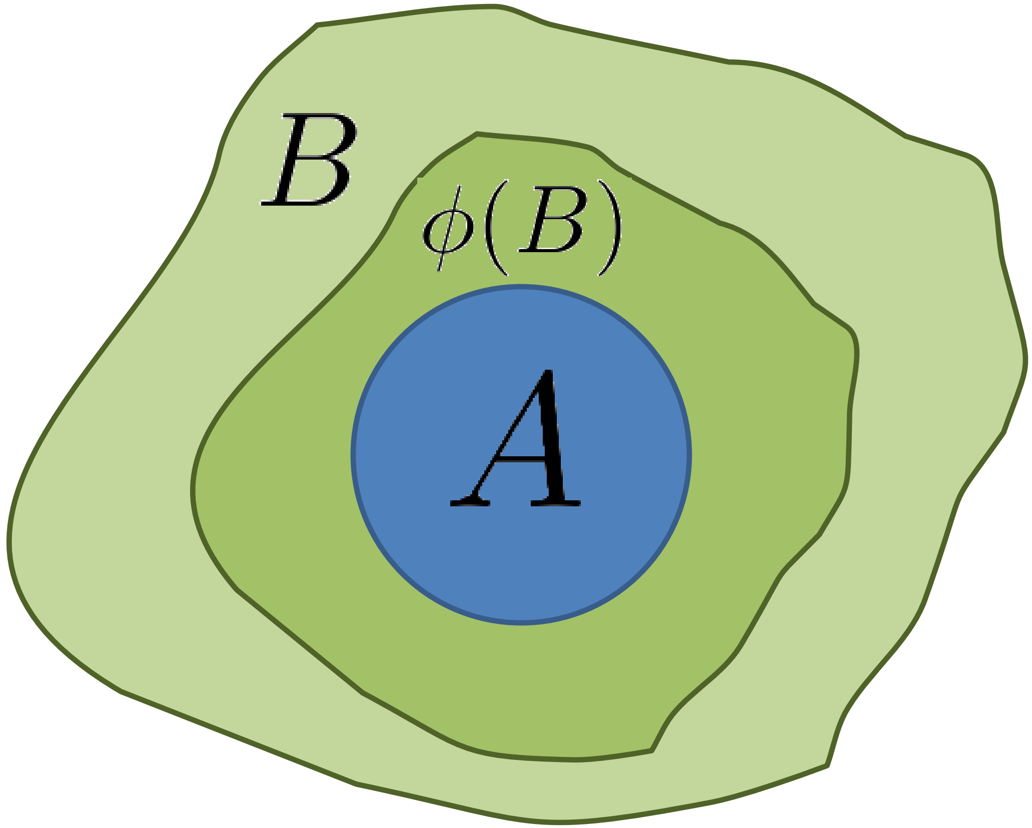
Definition: A is an **attractor** for ϕ if

1) A a nonempty, compact, invariant set, and

2) $\exists$ a neighborhood U of A such that $\omega(U) = A$

(McGehee 1988)

$\phi : X \circlearrowleft$



Definition:

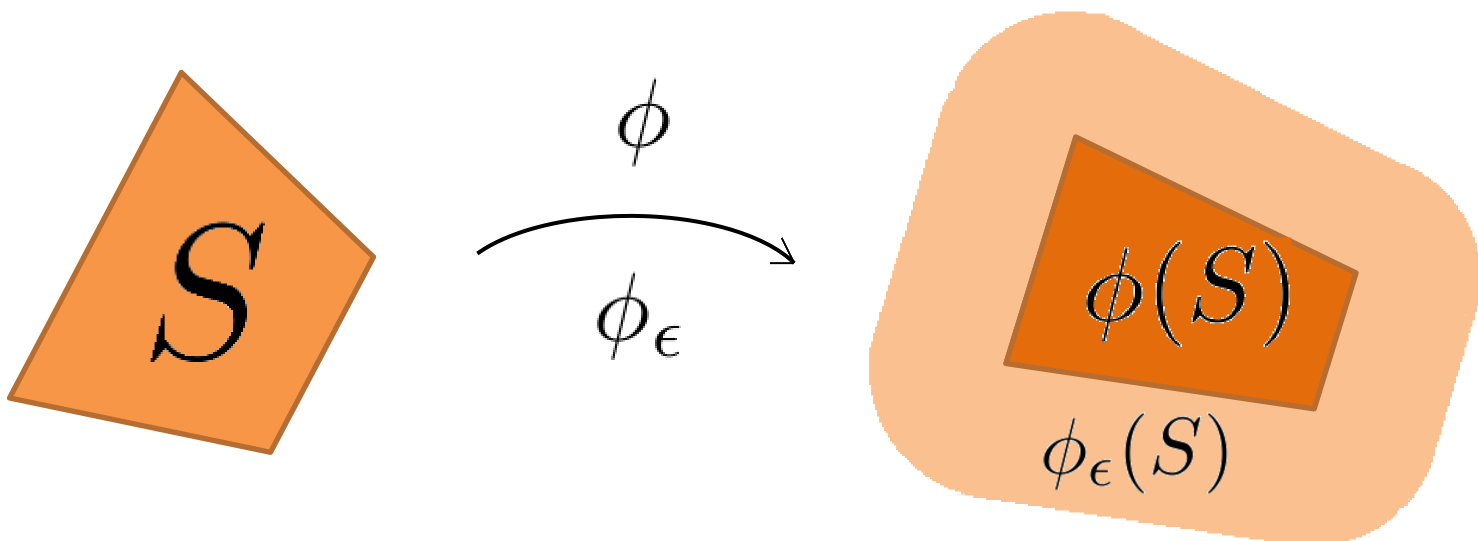
B is an **attractor block associated with** A if

1) B is compact and nonempty
2) $\phi(B) \subset B^o$
3) $\omega(B) = A$

(McGehee 1988)

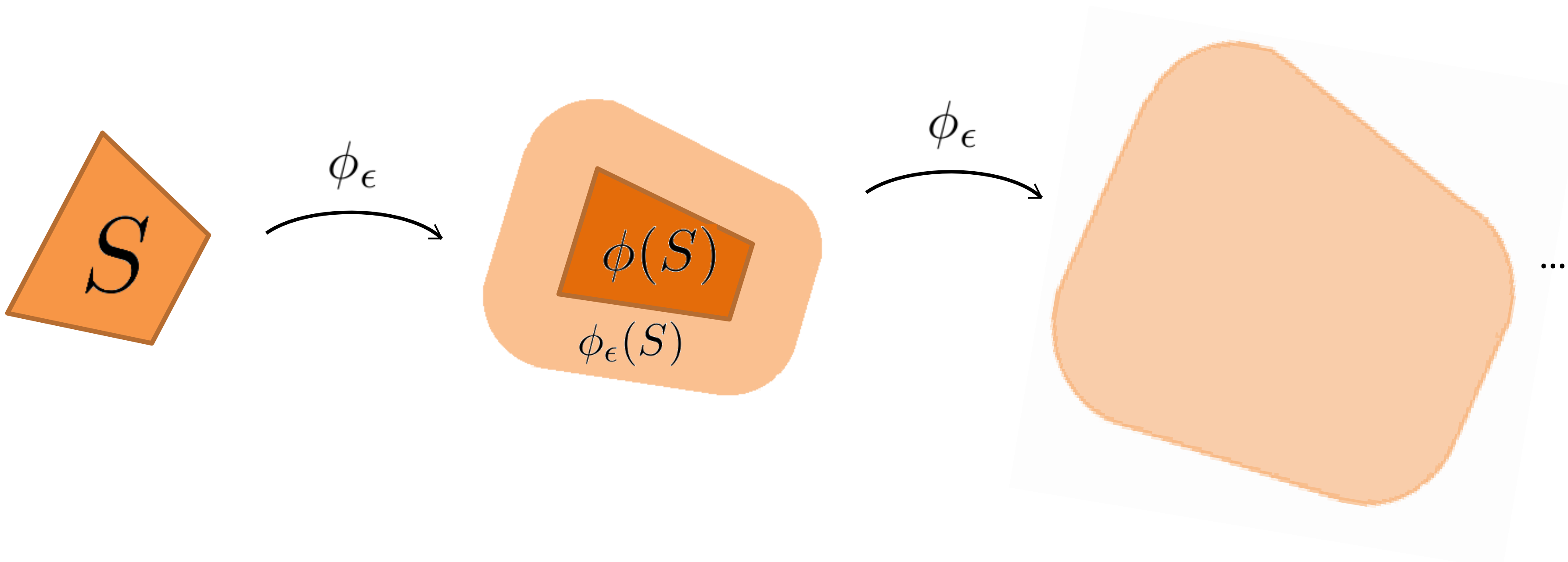
Define $\phi_\epsilon : \mathscr{P}(X) \to \mathscr{P}(X)$ by

$$\phi_\epsilon(S) = \{x \in X \mid \mathrm{dist}(x, \phi(S)) < \epsilon\}$$

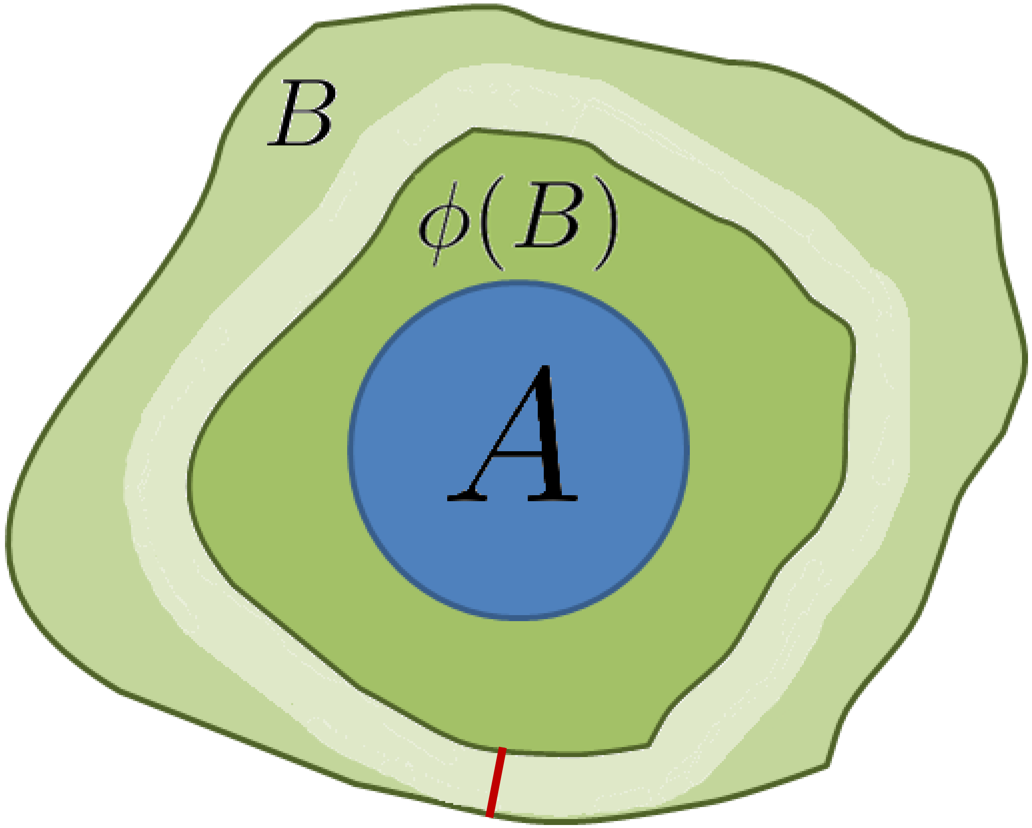


(McGehee 1988)

Let $P_\epsilon(S)$ denote the set of all points accessible by ϵ-pseudo-orbits starting on S



(McGehee 1988)



Definition: $\beta(B) \equiv \sup\{\epsilon \mid \phi_\epsilon(B) \subset B\}$

(McGehee 1988)

Definition: For an attractor A, the **intensity** of A is

$$\nu(A) \equiv \sup\{\beta(B) \mid B \text{ is an attractor block associated with } A\}$$

The **chain intensity** of A is

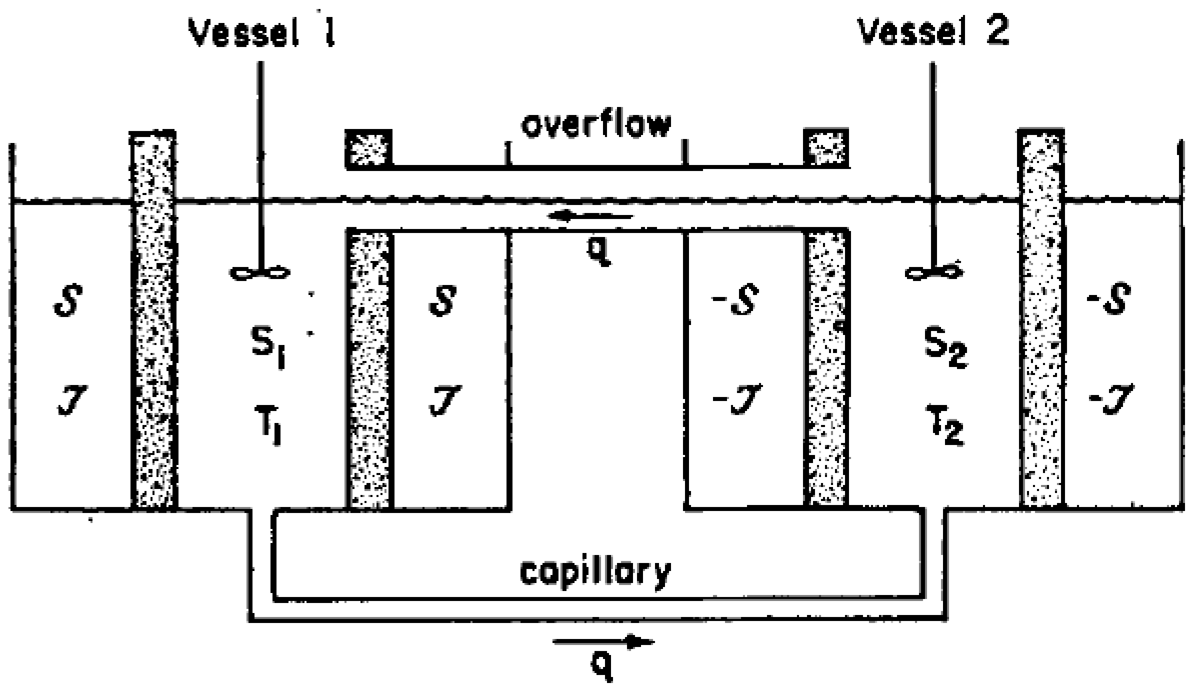
$$\mu(A) \equiv \sup\{\epsilon \mid P_\epsilon(A) \subset \text{ compact set } \subset \mathcal{D}(A)\}$$

Theorem: $\nu(A) \equiv \mu(A).$

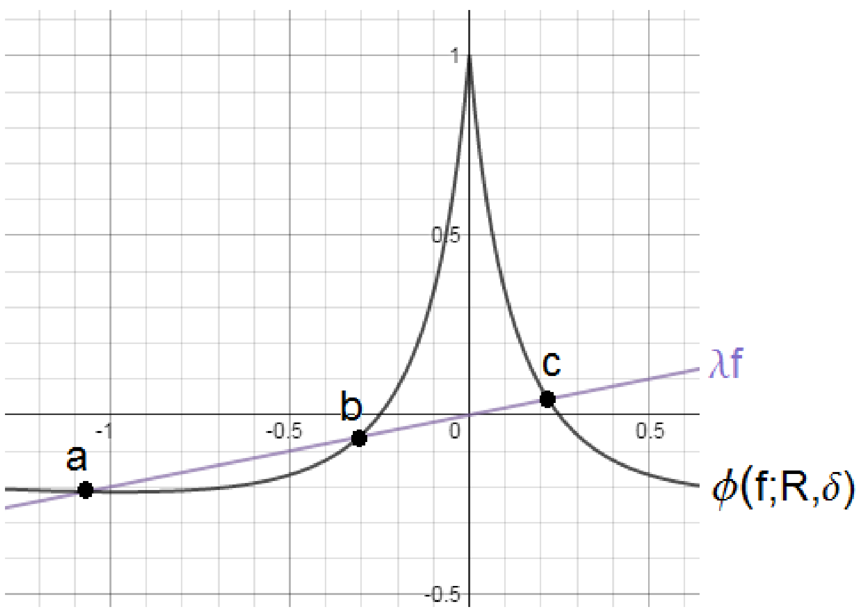
Question: *Do these quantities measure "resilience"?*

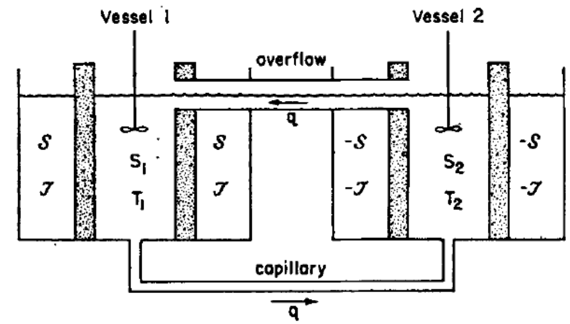
(McGehee 1988)

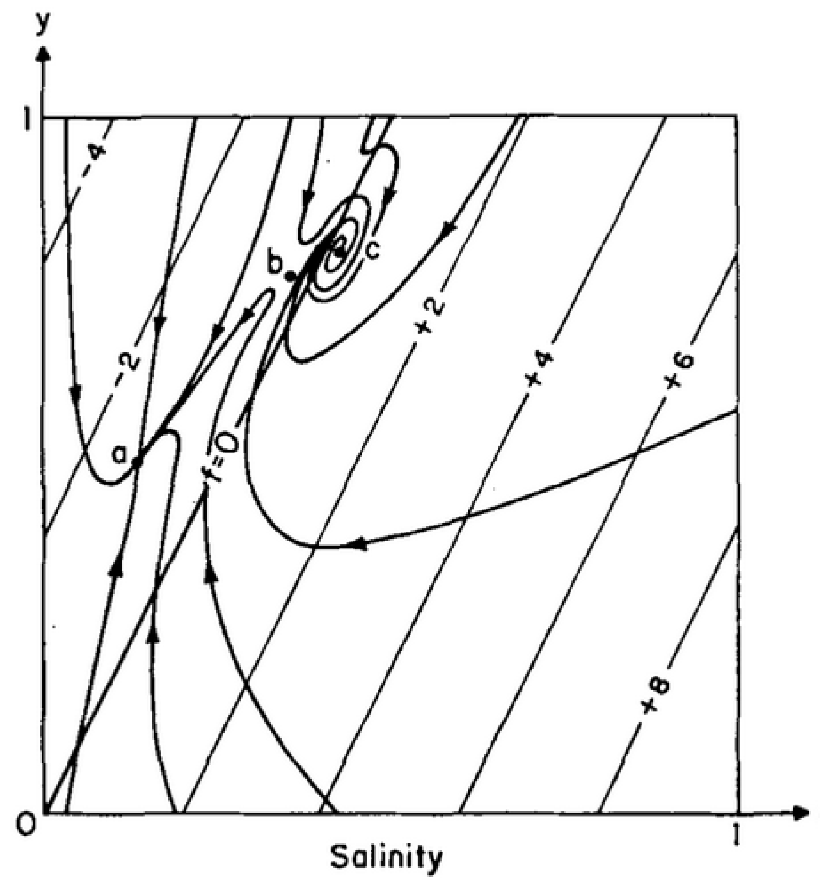
Idea: *Compute intensity of attraction in Stommel's ocean box model*



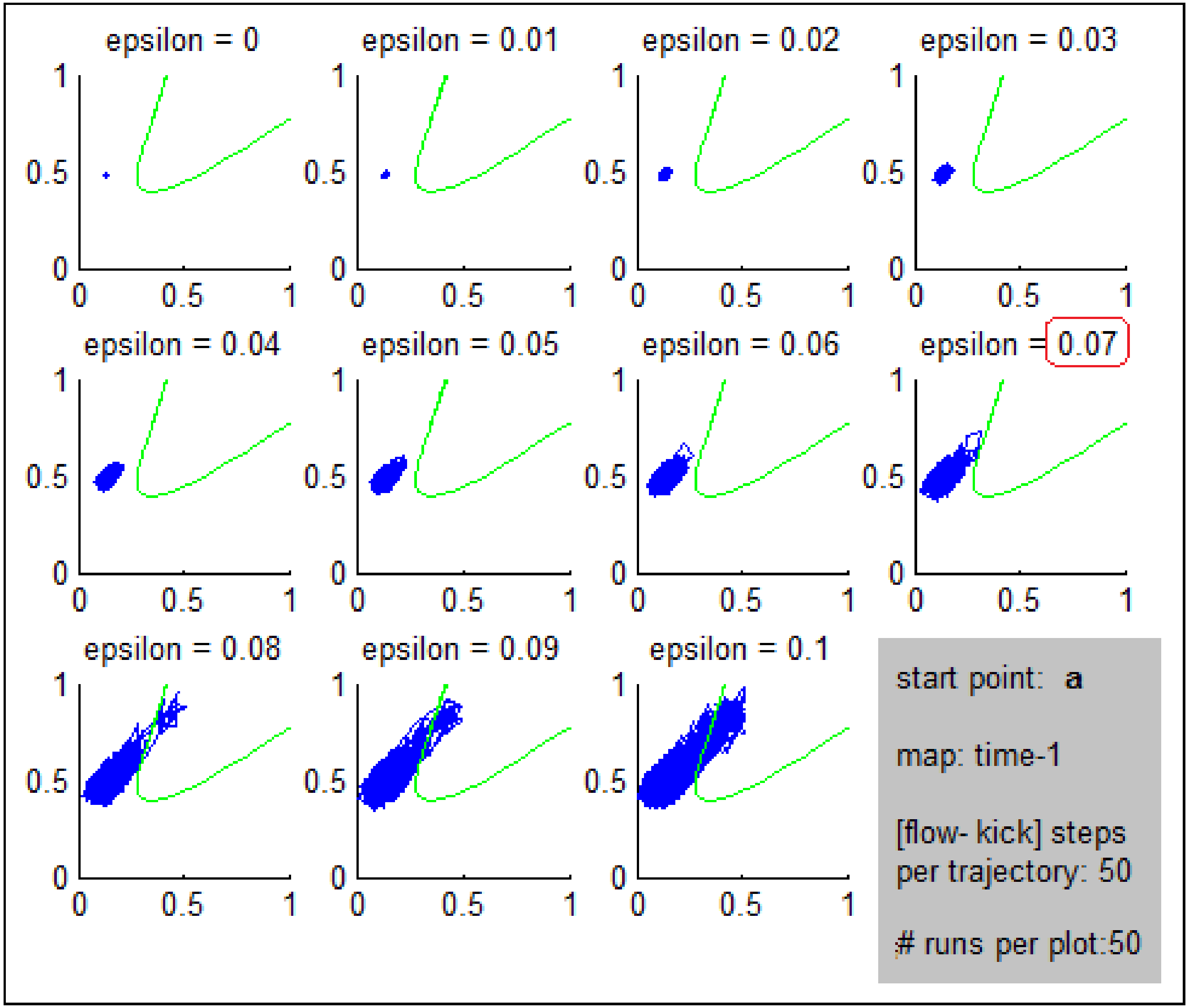
(Stommel 1961)



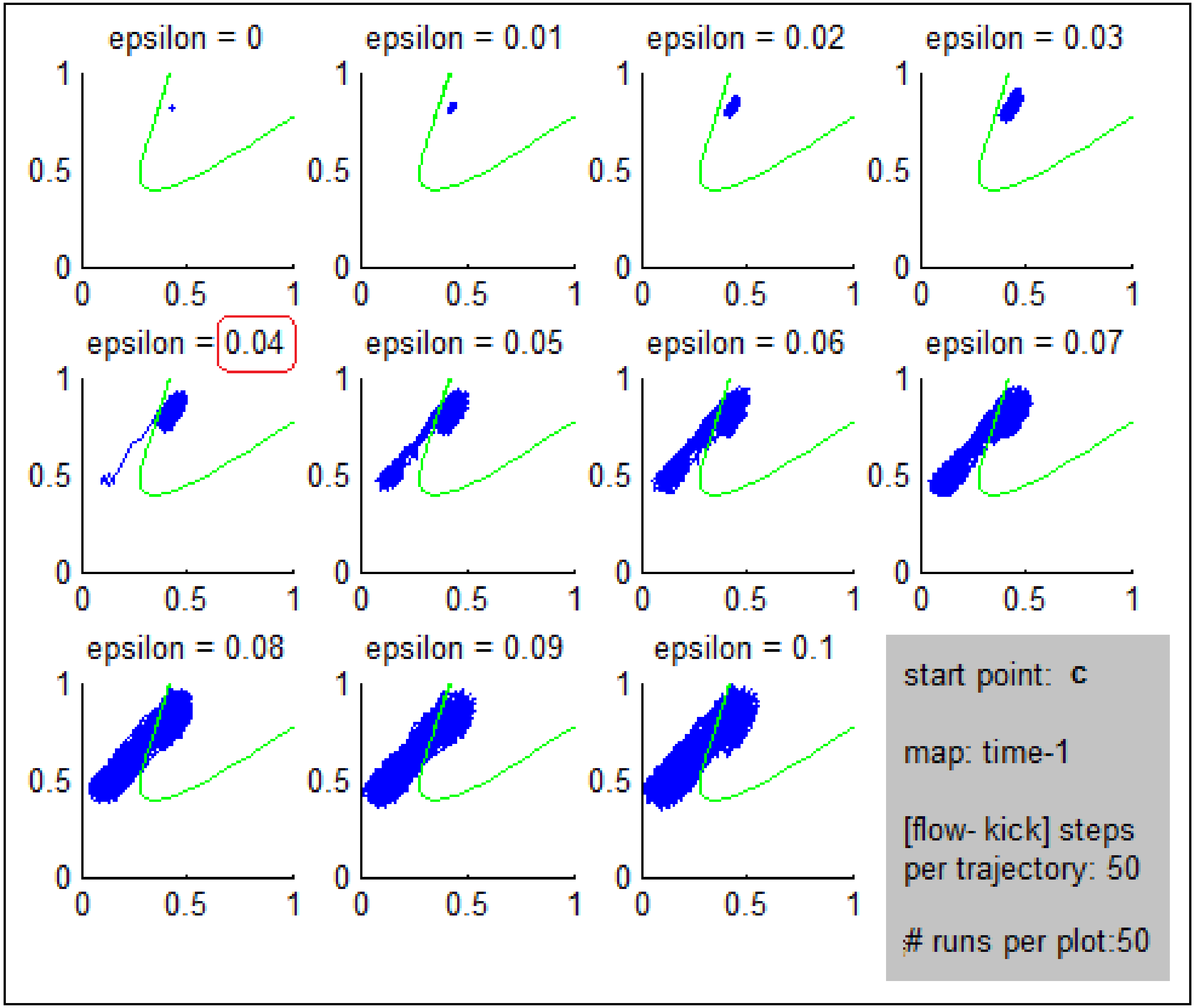




[Stommel 1961]



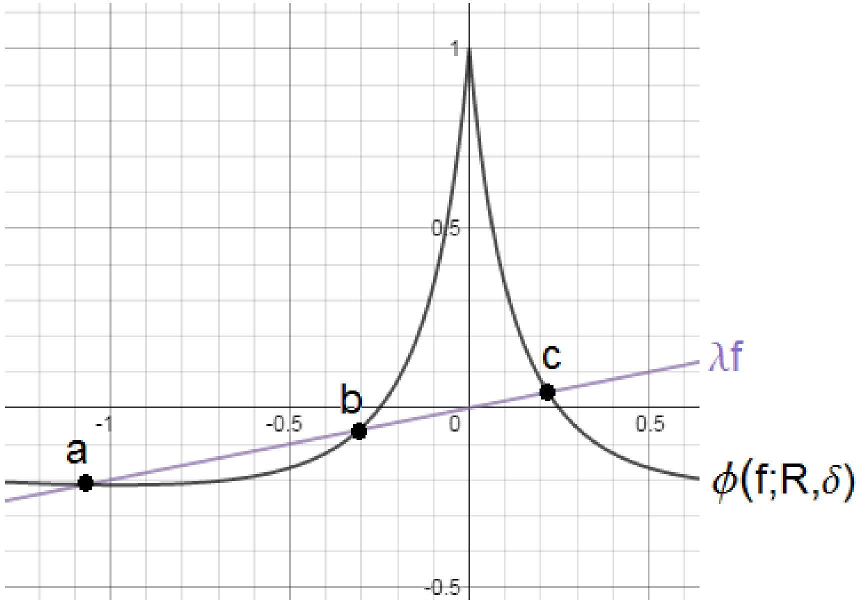
epsilon = 0
epsilon = 0.01
epsilon = 0.02
epsilon = 0.03
epsilon = 0.04
epsilon = 0.05
epsilon = 0.06
epsilon = 0.07
epsilon = 0.08
epsilon = 0.09
epsilon = 0.1
start point: a
map: time-1
[flow- kick] steps per trajectory: 50
runs per plot:50

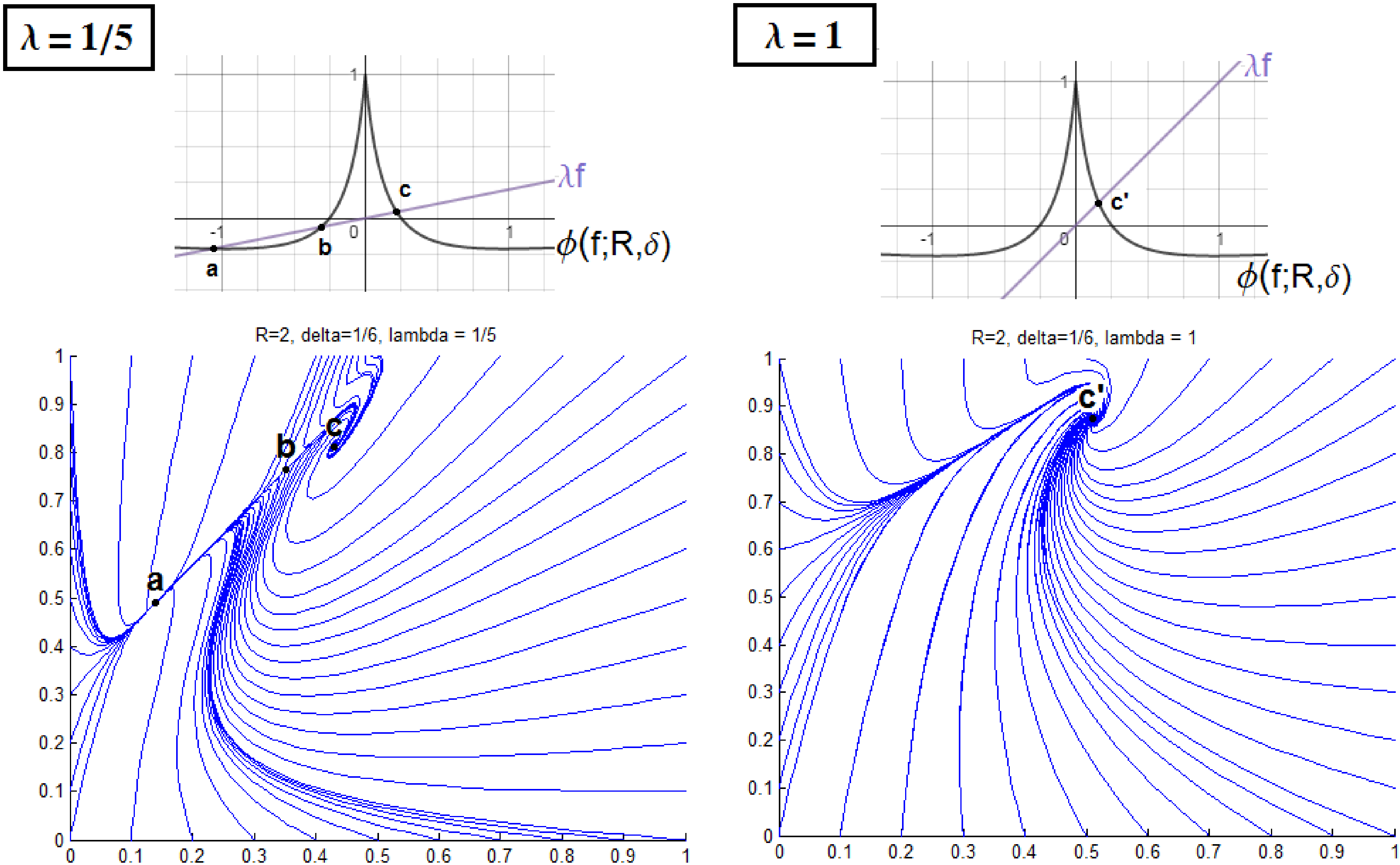

epsilon = 0
epsilon = 0.01
epsilon = 0.02
epsilon = 0.03
epsilon = 0.04
epsilon = 0.05
epsilon = 0.06
epsilon = 0.07
epsilon = 0.08
epsilon = 0.09
epsilon = 0.1
start point: c
map: time-1
[flow- kick] steps per trajectory: 50
runs per plot:50

Conclusion: When $\lambda = 0.2$,

$$\mu(\mathbf{a}) \approx 0.7 \quad \text{and} \quad \mu(\mathbf{c}) \approx 0.4.$$

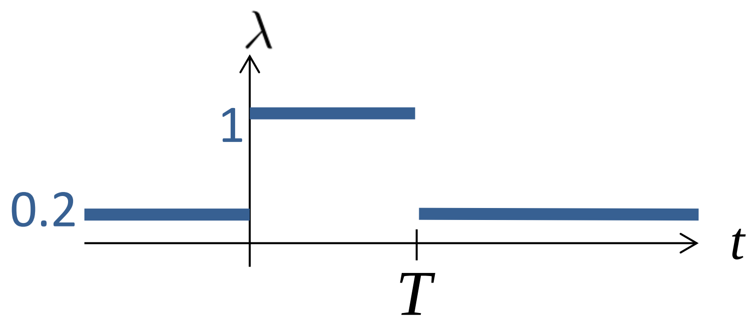
Is **a** *more resilient than* **c***?*



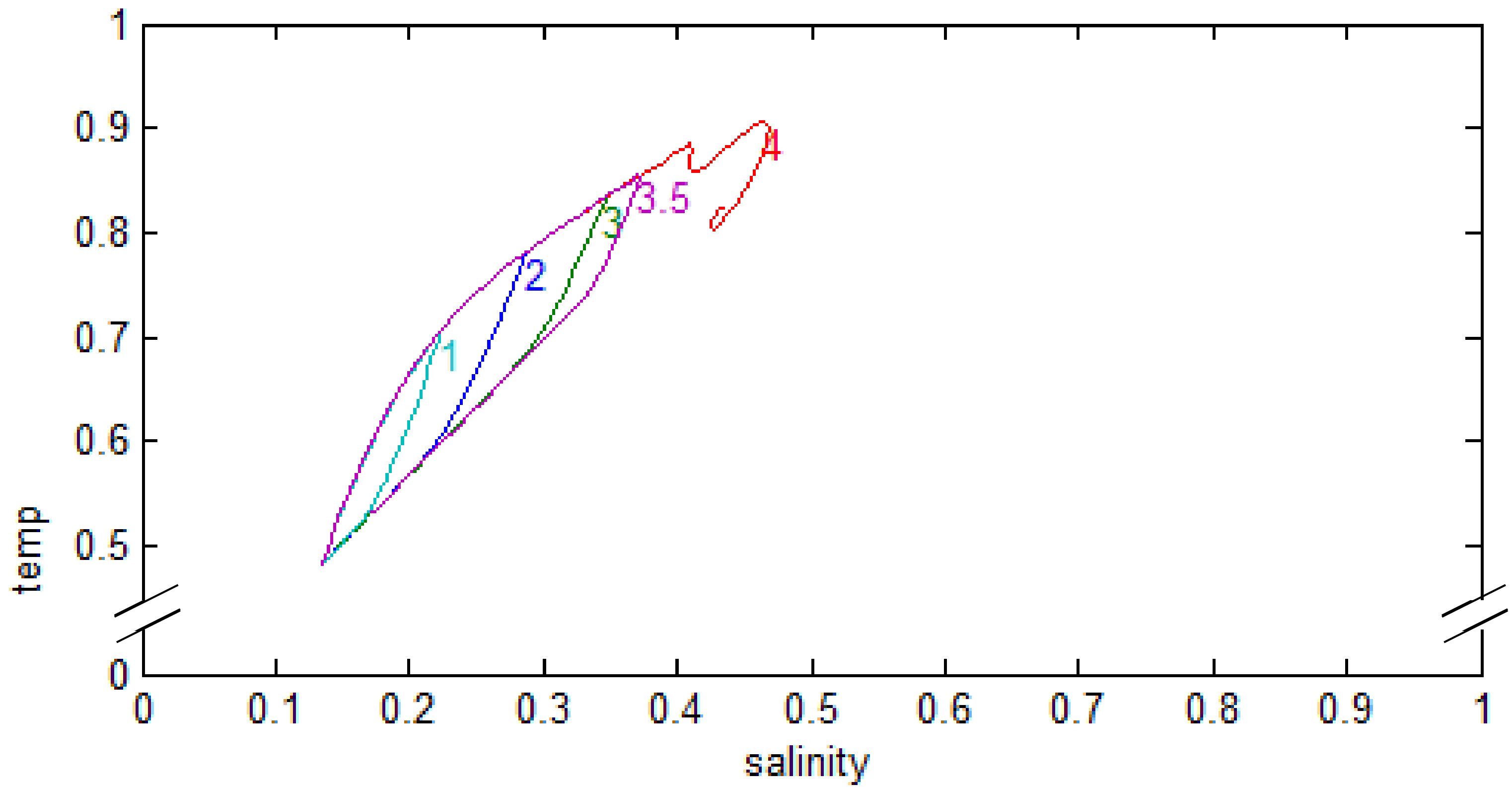


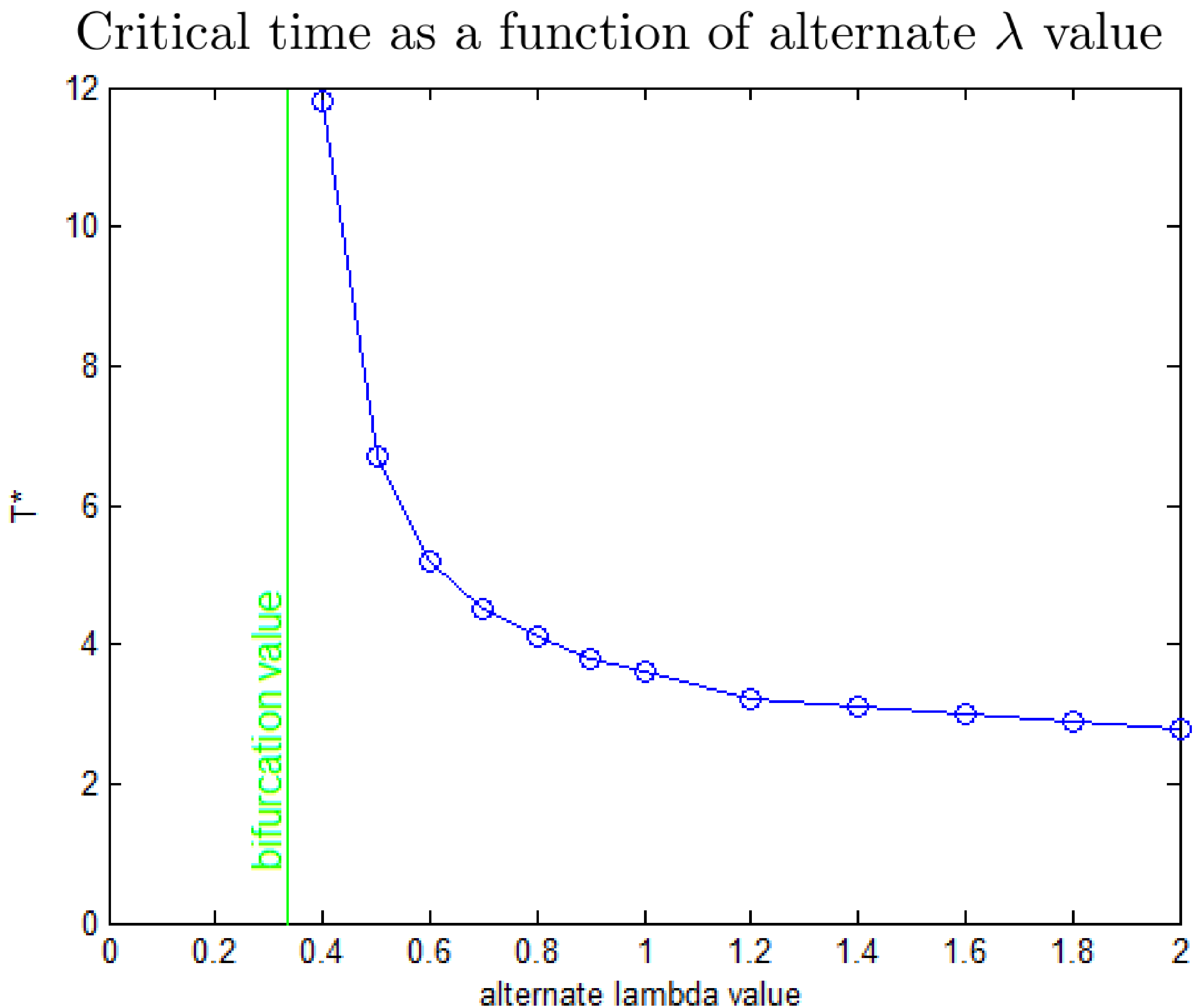
λ = 1/5
λf
φ(f;R,δ)
a
b
c
R=2, delta=1/6, lambda = 1/5
λ = 1
c'
R=2, delta=1/6, lambda = 1

Parameter perturbation schedule:



Trajectories for different values of T:

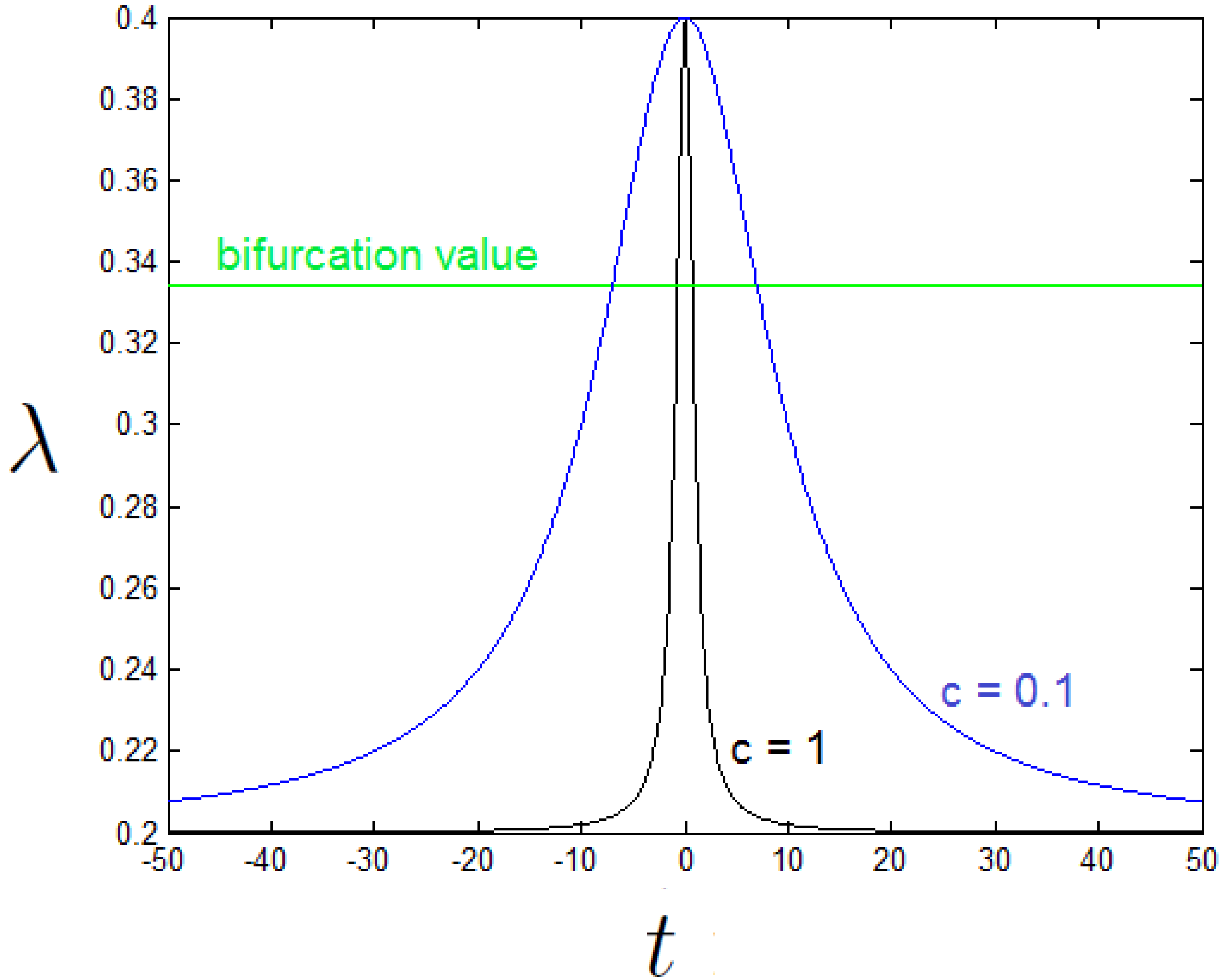


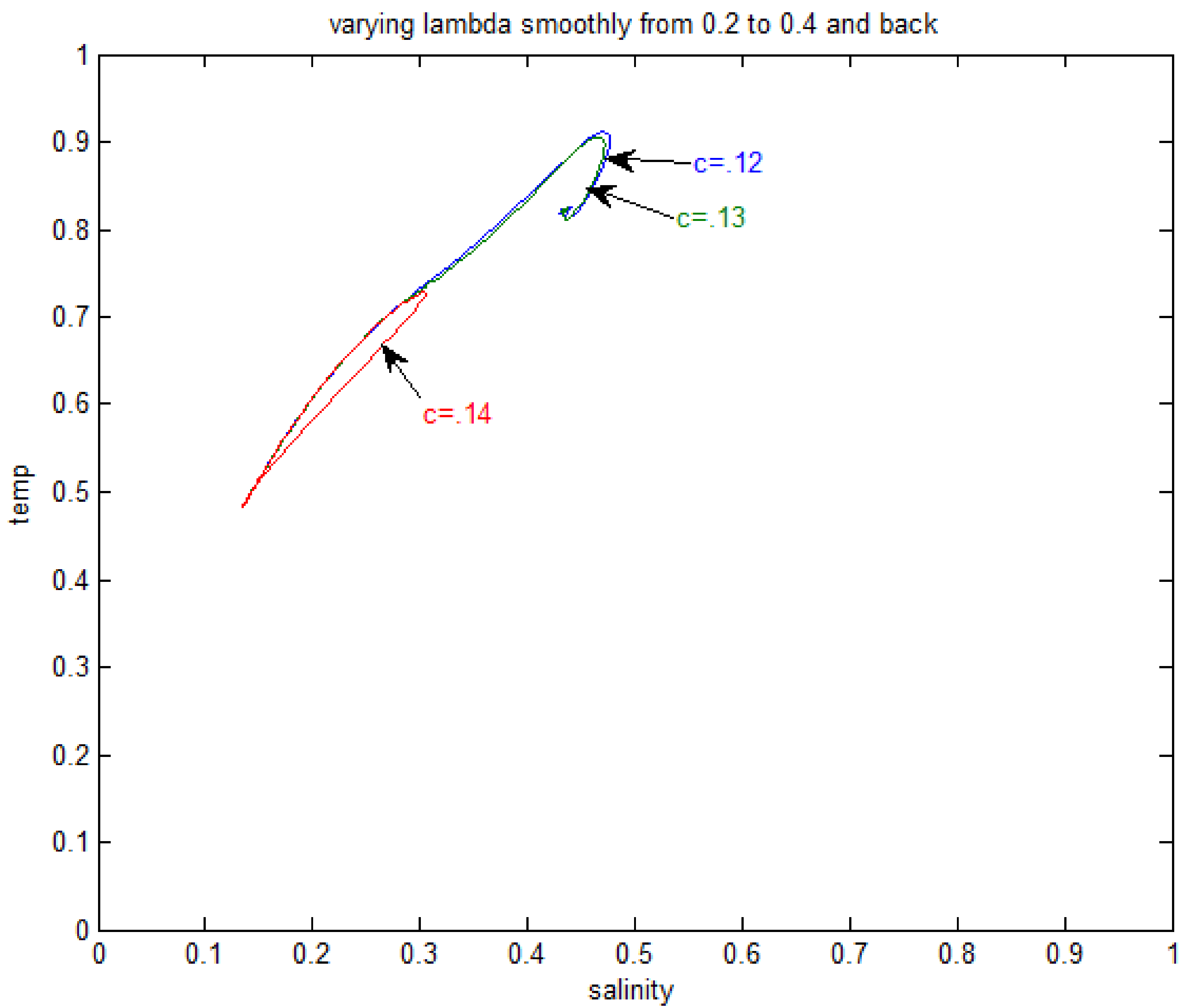

Critical time as a function of alternate λ value
12
10
8
6
4
2
0
T*
bifurcation value
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
alternate lambda value

Smooth variation of λ:

$$\lambda = \frac{0.2}{1+\mu^2} + 0.2$$

$$\mu = ct$$





varying lambda smoothly from 0.2 to 0.4 and back
temp
salinity
c=.12
c=.13
c=.14
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Future directions:

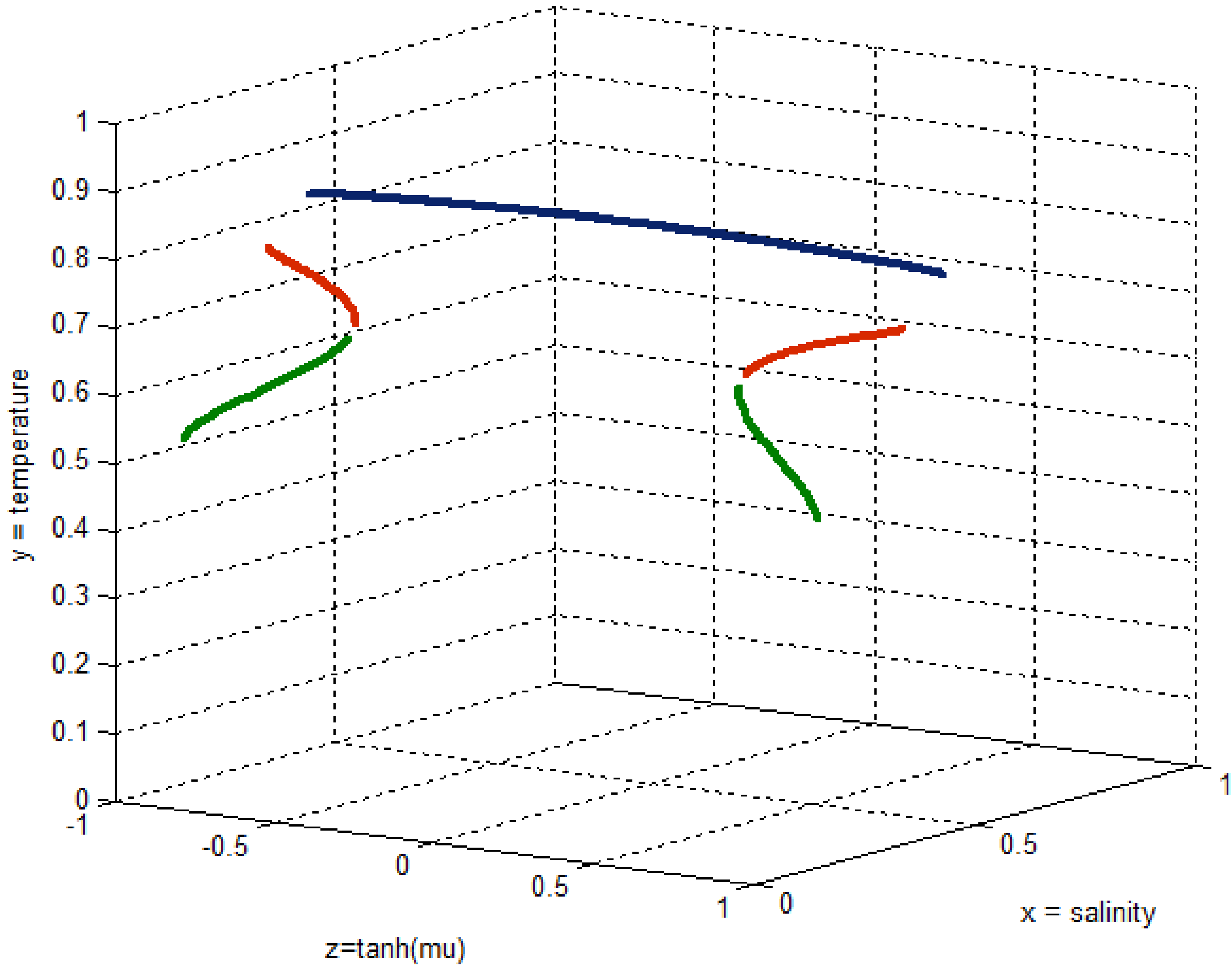
- Does "intensity of attraction" have an analogue for vector fields?
- For a given schedule of parameter perturbations, can we predict analytically whether the system returns to its initial basin of attraction?
- How does resilience to state variables changes relate to resilience to parameter changes?

References

McGehee, R. 1988. Some metric properties of attractors with applications to computer simulations of dynamical systems. Unpublished manuscript, 38 pp.

Stommel, H. 1961. Thermohaline convection with two stable regimes of flow, Tellus XIII, 2 pp. 224-230

Walker, B., C. S. Holling, S. R. Carpenter, and A. Kinzig. 2004. Resilience, adaptability and transformability in social–ecological systems. *Ecology and Society* 9(2): 5. [online] URL: http://www.ecologyandsociety.org/vol9/iss2/art5


y = temperature
x = salinity
z=tanh(mu)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.5
0
0.5
1
0
0.5
1